Lecture # 9

Wind driven circulation

Western boundary currents

oral examination Tuesday July 7., 2015

- 13:00 Tabea Kilchling
- 13:30 Isabell Hochfeld
- 14:00 Lucas Schmidt
- 14:30 Annika Buck
- 15:00 Elena Hirschhoff
- 15:30 Heninng Dorff
- 16:00 Jerome Sauer
- 16:30 Carolin Meier ?
- 17:00 Sophie Specht ?
- 17:30 Anna Wünsche ?

- momentum equation in planetary geostrophic approximation

$$f\boldsymbol{k}\times\boldsymbol{u} = -\boldsymbol{\nabla}_h p + \frac{\partial\boldsymbol{\tau}}{\partial z} + A_h\boldsymbol{\nabla}_h^2\boldsymbol{u} - R\,\boldsymbol{u}$$

  with stress vector $\boldsymbol{\tau}$ connecting to surface wind stress
  with lateral friction related to the lateral turbulent viscosity A_h
  and with turbulent Rayleigh (bottom) friction related to R
- vertically integrating from flat bottom to surface (assume $\boldsymbol{\tau}_b = 0$)

$$f\boldsymbol{k}\times\int_{-h}^{0}\boldsymbol{u}dz \;=\; -\int_{-h}^{0}\boldsymbol{\nabla}_h p dz + \boldsymbol{\tau}_a + \int_{-h}^{0}(A_h\boldsymbol{\nabla}_h^2 - R)\boldsymbol{u}dz$$

- for $h = const$ vertical integration and $\boldsymbol{\nabla}$ commute

$$f\boldsymbol{k}\times\boldsymbol{U} \;=\; -\boldsymbol{\nabla}_h\int_{-h}^{0} p dz + \boldsymbol{\tau}_a + (A_h\boldsymbol{\nabla}_h^2 - R)\boldsymbol{U}$$

  with transport $\boldsymbol{U} = \int_{-h}^{0}\boldsymbol{u}dz$
- use transport streamfunction ψ with $\boldsymbol{U} = \boldsymbol{k}\times\boldsymbol{\nabla}\psi$

$$-f\boldsymbol{\nabla}_h\psi = -\boldsymbol{\nabla}_h\int_{-h}^{0} p dz + \boldsymbol{\tau}_a + (A_h\boldsymbol{\nabla}_h^2 - R)\boldsymbol{k}\times\boldsymbol{\nabla}\psi$$

- vertically integrated momentum equation

$$-f\boldsymbol{\nabla}_h\psi = -\boldsymbol{\nabla}_h\int_{-h}^{0} p dz + \boldsymbol{\tau}_a + (A_h\boldsymbol{\nabla}_h^2 - R)\boldsymbol{k}\times\boldsymbol{\nabla}\psi$$

  with transport $\boldsymbol{U} = \int_{-h}^{0}\boldsymbol{u}dz$ and streamfunction $\boldsymbol{U} = \boldsymbol{k}\times\boldsymbol{\nabla}\psi$
- now take curl $(\boldsymbol{k}\times\boldsymbol{\nabla})\cdot$ of momentum equation

  $-(\boldsymbol{k}\times\boldsymbol{\nabla})\cdot f\boldsymbol{\nabla}_h\psi = \boldsymbol{k}\times\boldsymbol{\nabla}\cdot\boldsymbol{\tau}_a + (A_h\boldsymbol{\nabla}_h^2 - R)(\boldsymbol{k}\times\boldsymbol{\nabla})\cdot(\boldsymbol{k}\times\boldsymbol{\nabla})\psi$

  since $(\boldsymbol{k}\times\boldsymbol{\nabla})\cdot\boldsymbol{\nabla}_h\int_{-h}^{0} p dz = 0$
- with

  $-(\boldsymbol{k}\times\boldsymbol{\nabla})\cdot f\boldsymbol{\nabla}_h\psi = -f(\boldsymbol{k}\times\boldsymbol{\nabla})\cdot\boldsymbol{\nabla}_h\psi - \boldsymbol{\nabla}_h\psi\cdot(\boldsymbol{k}\times\boldsymbol{\nabla})f = \beta\dfrac{\partial\psi}{\partial x}$

  and with $(\boldsymbol{k}\times\boldsymbol{\nabla})\cdot(\boldsymbol{k}\times\boldsymbol{\nabla})\psi = \boldsymbol{\nabla}_h^2\psi$
- the Stommel/Munk equation for flat bottom follows as

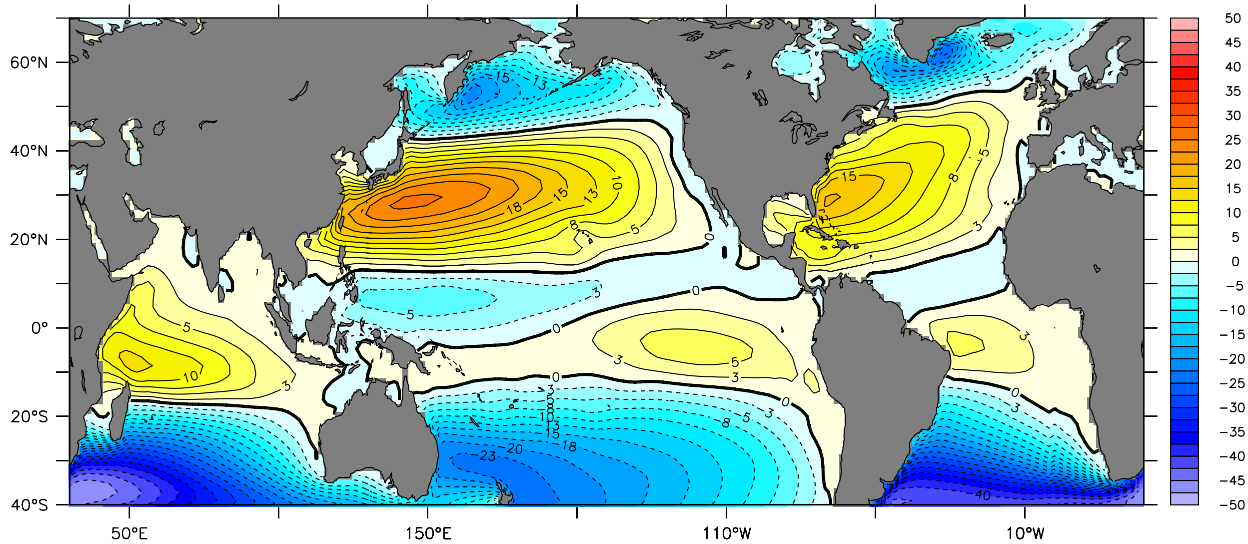
$$\beta\frac{\partial\psi}{\partial x} = \boldsymbol{k}\times\boldsymbol{\nabla}\cdot\boldsymbol{\tau}_a + (A_h\boldsymbol{\nabla}_h^2 - R)\boldsymbol{\nabla}_h^2\psi$$

  first part identical to Sverdrup relation, friction related to A_h and R closes circulation at western boundary

- numerical solution of Stommel's equation

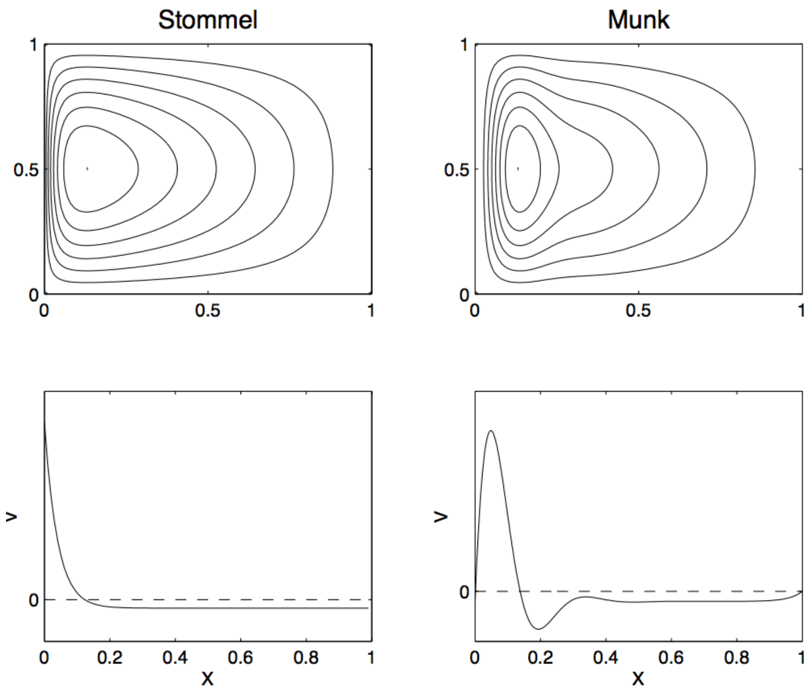
$$\beta \frac{\partial \psi}{\partial x} = \boldsymbol{k} \times \boldsymbol{\nabla} \cdot \boldsymbol{\tau}_a - R \boldsymbol{\nabla}_h^2 \psi$$

with realistic wind stress $\boldsymbol{\tau}_a$ (and $A_h = 0$)



- Henry Melson Stommel, * 1920 in Wilmington (USA) †1992 in Boston (USA), oceanographer.

- consider a wind stress of the form $\boldsymbol{\tau}_a = (-\tau_0 \cos \frac{\pi y}{B}, 0)$
  with $A_h = 0 \rightarrow$ Stommel's equation (left)
  and $R = 0 \rightarrow$ Munk's equation (right)



from Vallis 2006

- boundary layer scaling

$$\beta \frac{\partial \psi}{\partial x} = \boldsymbol{k} \times \boldsymbol{\nabla} \cdot \boldsymbol{\tau}_a + (A_h \boldsymbol{\nabla}_h^2 - R) \boldsymbol{\nabla}_h^2 \psi$$

- balance bottom friction $R\boldsymbol{\nabla}_h^2\psi$ and planetary vorticity $\beta\partial\psi/\partial x$

$$\begin{aligned} \beta\psi/L &\sim R\psi/L^2 \\ L &\sim R/\beta \end{aligned}$$

Stommel's boundary layer with $R/\beta \approx 50 - 100\,\mathrm{km}$

- balance lateral friction $A_h\boldsymbol{\nabla}_h^4\psi$ and planetary vorticity $\beta\partial\psi/\partial x$

$$\begin{aligned} \beta\psi/L &\sim A_h\psi/L^4 \\ L &\sim (A_h/\beta)^{1/3} \end{aligned}$$

Munk's boundary layer with $(A_h/\beta)^{1/3}$

- $(A_h/\beta)^{1/3}$ is often used to choose value for A_h in numerical models

- why is the western boundary current in the west?
- dominant balance in the western boundary current regime

$$\beta \frac{\partial \psi}{\partial x} = \beta V \approx \cancel{\boldsymbol{k} \times \boldsymbol{\nabla} \cdot \boldsymbol{\tau}_a} - R\boldsymbol{\nabla}_h^2\psi \approx -R\frac{\partial^2\psi}{\partial x^2} = -R\frac{\partial V}{\partial x}$$

between bottom friction and change in planetary vorticity

- since $V < 0$ in the interior of the subtropical gyre
  $V > 0$ in the western boundary $\rightarrow \beta V > 0 \rightarrow R\partial V/\partial x < 0$
- for western boundary layer V decreases to the east $\rightarrow \partial V/\partial x < 0$
- for eastern boundary layer V increases to the east
  $\rightarrow$ no eastern boundary layer
- westward Rossby waves propagate to the west
- they are reflected at the western boundary as short Rossby waves with eastward group velocity
- short Rossby waves are dissipated in the west and form the boundary current